Lecture 8
The Subjective Theory of *Betting on Theories*

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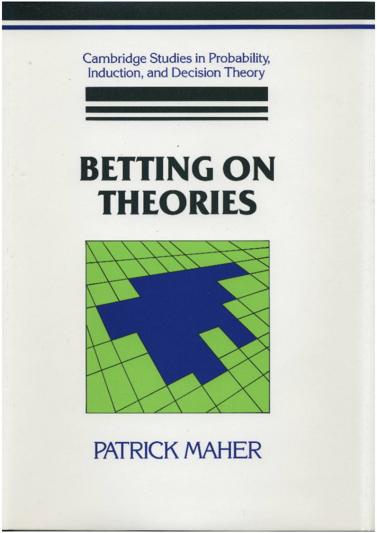
Philosophy 517
Spring 2007

Introduction

- The subjective theory of probability holds that the laws of probability are laws that rational degrees of belief must satisfy. Degrees of belief that satisfy these laws are called "subjective probabilities."
- Theories of inference or decision based on subjective probability are called "Bayesian."
- Subjective probability became popular in the 20th century.

The book



Cambridge Univ. Press 1993

On Google Books

Betting on Theories articulates and defends a particular version of the subjective theory of probability.

My view now: This is better than most subjective theories, but an objective theory is much better.

The Dutch book argument

This is the most popular argument that rational degrees of belief satisfy the laws of probability. I argued that it is fallacious.

Terminology and assumption (p. 95)

- *First some gambling terminology: If you pay \$r for the right to receive \$s if A is true, you are said to have made a bet on A with a* **betting quotient** *of r/s, and a* **stake** *s. Here r may be positive, zero, or negative; and s may be positive or negative.*
- *Dutch book arguments normally assume that for any proposition A, there is a number $p(A)$ such that you are willing to accept any bet with betting quotient $p(A)$. As the notation indicates, $p(A)$ is thought of as your subjective probability for A.*

Strategy (p. 95)

- *The task of the Dutch book argument is to show that . . . rationality requires p to satisfy the axioms of probability.*
- *Dutch book arguments purport to do this by showing that if p does not satisfy the axioms of probability, then you will be willing to accept bets that necessarily give you a sure loss. A set of bets with this property is called a* **Dutch book**.

Example of how this is supposed to work (p. 95)

Let H denote that a coin will land heads on the next toss, and suppose that for you $p(H) = p(\bar{H}) = .6$, *which violates the axioms of probability. The Dutch book argument, applied to this case, goes as follows: Since* $p(H) = .6$, *you are willing to pay 60 cents for a bet that pays you $1 if H. The same holds for* $\bar{H}$. *But these two bets together will result in you losing 20 cents no matter how the coin lands; they constitute a Dutch book. The Dutch book argument . . . concludes that your willingness to accept these two bets shows you are irrational.*

Example to show the fallacy (p. 96)

- *Suppose that, after a night on the town, you want to catch the bus home. Alas, you find that you have only 60 cents in your pocket, and the bus costs $1. A bookie, learning of your plight, offers you the following deal: If you give him your 60 cents, he will toss a coin; and if the coin lands heads, he will give you $1; otherwise, you have lost your 60 cents . . . You may well feel that the bookie's offer is acceptable; let us suppose you do. Presumably the offer would have been equally acceptable if you were betting on tails rather than heads.*
- *As subjective probability was defined for the simple Dutch book argument, your probability for heads in the above scenario is at least .6, and so is your probability for tails; thus your probabilities violate the probability calculus. The Dutch book argument claims to deduce from this that you are irrational. And yet . . . your preferences seem perfectly reasonable.*

The fallacy (p. 96)

- *From the fact that you are willing to accept each of two bets that together would give a sure loss, the argument infers that you are willing to give away money to a bookie. This assumes that if you are willing to accept each of the bets, you must be willing to accept both of them. But that assumption is surely false in the present case.*
- *This, then, is the fallacy in the Dutch book argument: It assumes that bets that are severally acceptable must also be jointly acceptable; and as our example shows, this is not so.*

I went on to show that other versions of the Dutch book argument either (a) do not avoid the fallacy, or (b) replace it with another fallacy, or (c) really abandon the Dutch book approach.

Representation theorem approach

Expected utility (p. 1)

- Let f be an act and $x_1, x_2, \ldots,$ the states of the world that might influence its outcome.
- Let o_i be the outcome f will have if x_i is the true state.
- Let p be a probability function defined on the x_i and u a utility function defined on the o_i.
- The *expected utility of f*, relative to p and u, is:
$$EU(f) = p(x_1)u(o_1) + p(x_2)u(o_2) + \cdots .$$

Representation theorems

A representation theorem shows that if a person's preferences satisfy certain qualitative conditions then there exists a probability function p and a utility function u such that, for all acts f and g, the person prefers f to g iff $EU(f) > EU(g)$.

Conditions assumed (p. 10)

In many representation theorems, including mine, the assumed conditions include:

1. **Connectedness.** For any acts f and g, you either prefer f to g, or prefer g to f, or are indifferent between them.
2. **Transitivity.** If you weakly prefer f to g, and weakly prefer g to h, then you weakly prefer f to h.
    - To weakly prefer f to g means that you either prefer f to g or else are indifferent between them.
3. **Independence.** If two acts have the same consequence in some states, then your preference between those acts is independent of what that common consequence is.

Independence example (pp. 63–64)

A ball is to be randomly drawn from an urn containing 100 balls numbered from 1 to 100. Consider the following options:

	1	2–11	12–100
a_1	\$1 million	\$1 million	\$1 million
a_2	\$0	\$5 million	\$1 million
b_1	\$1 million	\$1 million	\$0
b_2	\$0	\$5 million	\$0

If the money you get is all that matters, and you prefer a_1 to a_2, then independence requires you to prefer b_1 to b_2.

Simple argument (p. 9)

1. A rational person's preferences satisfy connectedness, transitivity, independence, etc.
2. So, for a rational person, there exists a probability function p and a utility function u such that the person always prefers acts with higher expected utility relative to p and u. (The representation theorem shows this follows.)
3. If a person always prefers acts with higher expected utility relative to p and u, then p correctly measures the person's degrees of belief and u correctly measures the person's values.
4. Therefore, a rational person has degrees of belief that satisfy the laws of probability.

Contrasts with the Dutch book argument

On the representation theorem approach:

- If you are willing to pay 60 cents for a bet that pays $1 if H, it does not follow that your degree of belief in H is at least .6; your utilities need to be taken into account.
- It is not assumed that separately acceptable acts are jointly acceptable. The combination of two acts is a third act with its own expected utility.

Questions

1. What is a Dutch book? What is the simple Dutch book argument? Is this argument sound? Why, or why not?
2. What does a representation theorem say? Describe three conditions that are often assumed in a representation theorem.
3. State the simple representation-theorem argument for the conclusion that a rational person has degrees of belief that satisfy the laws of probability.

Relaxing the conditions

Rationality

- There are different concepts of rationality; we saw some of them in Lecture 1.
- In *Betting on Theories* I took an act to be rational if it was advisable for the person to choose it, all things considered. This is *practical rationality*.
- A *state*, such as a belief or a preference, is rational in this sense if it would be advisable for the person to do what it takes to be in that state.
- But then, the conditions of the representation theorem are not always requirements of rationality.

Transitivity and independence (p. 12)

- *I do not hold that rationality always forbids violations of transitivity or independence. For example, an anti-Bayesian tycoon might offer me a million dollars to have preferences that are intransitive in some insignificant way; then I would agree that if I could make my preferences intransitive, this would be the rational thing for me to do. More realistically, it may be that my preferences are intransitive, but it would take more effort than it is worth to remove the intransitivities, in which case the rational thing to do would be to remain intransitive.*
- *What I do hold is that when the preferences are relevant to a sufficiently important decision problem, and there are no rewards attached to violating transitivity or independence, then it is rational to have one's preferences satisfy these conditions.*
- *I will make it a standing assumption that we are dealing with situations in which the conditions [just stated] are satisfied.*

Connectedness

- *Everyone must have had the experience of agonizing over a decision, not knowing what to do. In such a situation, it seems most natural to say that we neither prefer one option to the other nor are indifferent between them. But then we violate ... connectedness.* (p. 19)
- *Besides the difficult decisions we actually face, there is a huge class of difficult decisions we have not had to face. In many cases, we also lack preferences about the options in these hypothetical decision problems. However, ... connectedness ... requires that one have preferences, even about merely hypothetical options ... It is hard to see how rationality could require you to have preferences ... about all the merely hypothetical options that are not available to you.* (p. 19)
- *I then have to say that the connectedness postulate is not a requirement of rationality.* (p. 20)

Representation by sets

- *A more plausible condition is that rationality requires your preferences, so far as they go, to agree with at least one connected preference ordering that satisfies transitivity, independence, and the other assumptions of a representation theorem . . . I will adopt this condition.* (p. 20)
- *We can then regard your unconnected preferences as represented by the set of all pairs of probability and utility functions that represent a connected extension of your preferences. I will call this set your representor and its elements p-u pairs.* (p. 21)
- *At the beginning . . . I described Bayesian decision theory as holding that rational persons have a probability and utility function . . . A more explicit statement of the position I am defending would be that a rational person has a representor that is nonempty; that is, it contains at least one p-u pair.* (p. 21)

Correction

- Vague degrees of belief have vague boundaries, and that isn't always irrational. Hence rational people needn't have any definite representor.
- *Where the error crept in:* I assumed that for every connected preference ordering, your preferences either agree with it or they don't; that won't be true when preferences are vague.
- *How to fix this:* Just say that a rational person's degrees of belief are consistent with at least one probability function.

The final, corrected, claim ("BOT's claim")

When a sufficiently important decision problem is involved, and there are no rewards attached to violating the conditions of a representation theorem, then practically rational degrees of belief are consistent with at least one probability function.

An objective alternative

What it says

- Instead of talking about *practically rational degrees of belief*, we talk about *inductive probabilities*. We dispense with the psychological concept of belief.
- In place of BOT's claim, we say: *Inductive probabilities are consistent with at least one probability function.*

BOT's claim is correct

- When a sufficiently important decision problem is involved, and there are no rewards attached to violating the conditions of a representation theorem, then it is practically rational to have one's degrees of belief agree with the inductive probabilities given one's evidence.
- Therefore, BOT's claim is correct.

The objective theory is much better, for the following reasons.

Generality

- BOT's claim must be restricted to situations in which "a sufficiently important decision problem is involved, and there are no rewards attached to violating the conditions of a representation theorem." The objective theory doesn't require this or any other restriction.
- That's important because probability theory is used to evaluate the confirmation of theories by evidence in contexts where no specific decisions are envisaged.

Simplicity

- The representation theorem is complex—it contains many additional conditions that haven't been stated here. The proof is also complex—see the Appendices of *Betting on Theories*.
- The objective theory doesn't require a representation theorem. The claim about inductive probabilities is justified by direct examination.

Plausibility

- The representation theorem argument assumes a connection between degrees of belief and preference that is controversial. The theorem also makes numerous assumptions that are not obviously requirements of rationality.
- The objective theory just makes one simple and intuitive claim; it is much more likely to be right.

Why subjectivists won't buy this

- Subjectivists don't believe in "logical probability," hence not in inductive probability. That was my position too in 1993.
- But that skepticism is based on misunderstandings. We examined many skeptical arguments in Lectures 6 and 7.

Other subjective theories

We've examined one subjective theory; there are many others. A general method for dealing with them:

1. What kind of rationality are they talking about?
   - If practical rationality, the claim is false without a restriction like the one in BOT. Nobody else imposes that restriction.
   - If instrumental epistemic rationality, the claim is false without a similar restriction, which nobody imposes.
   - If evidential rationality, what does that mean, if not inductive probability given the person's evidence? This abandons subjectivism.
   - If logical consistency, the claim is false because degrees of belief can't be logically inconsistent.
2. Do they say that rational degrees of belief are representable by a set of probability functions? Then they are wrong because vague degrees of belief don't have precise boundaries.

This rules out every other subjectivist theory known to me.

Questions

4. What did *Betting on Theories* end up claiming about rational degrees of belief? (You can give the corrected version.)
5. State an objective alternative to the subjective theory of *Betting on Theories*. Which is better, and why?
6. James Joyce (2004 pp. 135–136) presents an argument that "it is practically irrational to hold beliefs that violate the laws of probability." Explain how you can tell, without looking at the argument, that it is unsound.
7. Mark Kaplan (1996 ch. 1) argued that if your degrees of belief are epistemically rational then they are representable by a set of probability functions. Explain how you can tell, without looking at his argument, that it is unsound.

References

- Joyce, James. 2004. Bayesianism. In *The Oxford Handbook of Rationality*, ed. Alfred R. Mele and Piers Rawling, 132–155. Oxford University Press.
- Kaplan, Mark. 1996. *Decision Theory as Philosophy*. Cambridge University Press.
- Maher, Patrick. 2006. Review of *Putting Logic in its Place: Formal Constraints on Rational Belief*, by David Christensen. *Notre Dame Journal of Formal Logic* 47: 133–149. (Section 5 criticizes Kaplan's argument that rational degrees of belief can be represented by a set of probability functions.)