Lecture 6
Keynes's Logical Theory

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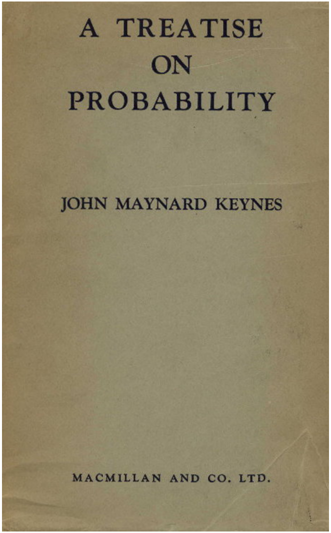
The book



Published 1921
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The subject matter of this book was first broached in the brain of Leibniz, who, in the dissertation, written in his twenty-third year, on the mode of electing the kings of Poland, conceived of Probability as a branch of Logic. (p. v)

The meaning of probability

Quotation from Keynes (pp. 5–6)

In the ordinary course of thought and argument, we are constantly assuming that knowledge of one statement, while not proving *the truth of a second, yields nevertheless* some ground *for believing it . . . And it does not seem on reflection that the information we convey by these expressions is wholly subjective . . . We are claiming, in fact, to cognise correctly a logical connection between one set of propositions which we call our evidence and which we suppose ourselves to know, and another set which we call our conclusions, and to which we attach more or less weight according to the grounds supplied by the first . . . It is not straining the use of words to speak of this as the relation of probability. It is true that mathematicians have employed the term in a narrower sense; for they have often confined it to the limited class of instances in which the relation is adapted to an algebraic treatment. But in common usage the word has never received this limitation.*

Keynes's probability is ip

- It is a sense that "probability" has in ordinary language.
- It is relative to evidence.
- It is objective, not subjective.
- It is not always quantifiable.

Measurement of probabilities

Some probabilities aren't quantifiable (p. 28)

We are out for a walk—what is the probability that we shall reach home alive? Has this always a numerical measure? If a thunderstorm bursts upon us, the probability is less than it was before; but is it changed by some definite numerical amount? There might, of course, be data that would make these probabilities numerically comparable; it might be argued that a knowledge of the statistics of death by lightning would make such a comparison possible. But if such information is not included within the knowledge to which the probability is referred, this fact is not relevant to the probability actually in question and cannot affect its value.

. . . even when statistics are available (p. 28)

In some cases, moreover, where general statistics are available, the numerical probability which might be derived from them is inapplicable because of the presence of additional knowledge with regard to the particular case. Gibbon calculated his prospects of life from the vital statistics and the calculations of actuaries. But if a doctor had been called to his assistance the nice precision of these calculations would have become useless; Gibbon's prospects would have been better or worse than before, but he would no longer have been able to calculate to within a day or week the period for which he then possessed an even chance of survival.

Some aren't even comparable (pp. 28–29)

Consider [two] sets of experiments, each directed towards establishing a generalisation. The first set is more numerous; in the second set the irrelevant conditions have been more carefully varied . . . Which of these generalisations is on such evidence the most probable? There is, surely, no answer; there is neither equality nor inequality between them . . . If we have more grounds than before, comparison is possible; but, if the grounds in the two cases are quite different, even a comparison of more and less, let alone numerical measurement, may be impossible.

Example (by me)

Generalization: All crows are black. Experiments:

- 1000 crows in Illinois are examined; all are found to be black.
- 500 crows are examined, half from Illinois and half from California; all are found to be black.

Existence of inductive probabilities

Ramsey's argument from nonagreement (Ramsey 1926, p. 10)

But let us now return to a more fundamental criticism of Mr. Keynes' views, which is the obvious one that there really do not seem to be any such things as the probability relations he describes. He supposes that, at any rate in certain cases, they can be perceived; but speaking for myself I feel confident that this is not true. I do not perceive them, and if I am to be persuaded that they exist it must be by argument; moreover, I shrewdly suspect that others do not perceive them either, because they are able to come to so very little agreement as to which of them relates any two given propositions.

My response: There is lots of agreement

- Practically every one agrees that the probability that a ball is white, given only that it is either black or white, is 1/2.
- There are zillions of other examples.

Ramsey's argument from simple propositions (Ramsey p. 10)

It is true that about some particular cases there is agreement, but these somehow paradoxically are always immensely complicated; we all agree that the probability of a coin coming down heads is $\frac{1}{2}$, be we can none of us say exactly what is the other term for the probability relation about which we are then judging. If, on the other hand, we take the simplest possible pairs of propositions such as "This is red" and "That is blue" or "This is red" and "That is red," whose logical relations should surely be easiest to see, no one, I think, pretends to be sure what is the probability relation which connects them.

My response: There is agreement in simple cases

- The example I just gave was simple.
- In Ramsey's examples, nobody pretends to be sure of a *numerical value* for the probabilities; but that's *agreement*, not disagreement.

My argument for existence

1. Ip's exist iff some sentences, which assert that an ip has a particular value, are true.
2. Consider the sentence:

   WB. The probability that a ball is white, given that it is either white or black, is $1/2$.

   This is an elementary statement of ip, hence either analytic or contradictory.
3. Since most people agree that WB is true, WB is probably not contradictory, but rather analytic.
4. Therefore, WB is true and ip's exist.

Questions

1. What did Keynes mean by "probability"? Justify your answer
2. Give an example of probabilities that Keynes would say are (a) comparable but not quantifiable, and (b) not comparable.
3. How did Ramsey argue that "there really do not seem to be any such things as the probability relations [Keynes] describes"? Evaluate his argument.
4. What is Maher's argument that inductive probabilities exist?

The principle of indifference

Terminology (p. 41)

In order that numerical measurement may be possible, we must be given a number of equally *possible alternatives. The discovery of a rule, by which equiprobability could be established, was, therefore, essential. A rule, adequate to that purpose . . . has been widely adopted, generally under the title of* The Principle of Non-Sufficient Reason . . . *This description is clumsy and unsatisfactory, and, if it is justifiable to break away from tradition, I prefer to call it* The Principle of Indifference.

Statement of the principle (p. 42)

The Principle of Indifference asserts that if there is no known *reason for predicating of our subject one rather than another of several alternatives, then relatively to such knowledge the assertions of each of these alternatives have an* equal *probability.*

Application to contradictories (pp. 42–43)

Consider a proposition, about the subject of which we know only the meaning, and about the truth of which, as applied to this subject, we possess no external relevant evidence. It has been held that there are here two exhaustive and exclusive alternatives—the truth of the proposition and the truth of its contradictory—while our knowledge of the subject affords no ground for preferring one to the other. Thus if a and $\bar{a}$ are contradictories, about the subject of which we have no outside knowledge, it is inferred that the probability of each is 1/2. In the same way the probabilities of two other propositions, b and c, having the same subject as a, may be each 1/2. But without having any evidence bearing on the subject of these propositions we may know that the predicates are contraries amongst themselves, and, therefore, exclusive alternatives—a supposition which leads by means of the same principle to values inconsistent with those just obtained.

Book example (p. 43)

If, for instance, having no evidence relevant to the colour of this book, we could conclude that 1/2 is the probability of 'This book is red,' we could conclude equally that the probability of each of the propositions 'This book is black' and 'This book is blue' is also 1/2. So that we are faced with the impossible case of three *exclusive alternatives all as likely as not.*

Application to specific volume (p. 45)

Consider the specific volume of a given substance. Let us suppose that we know the specific volume to lie between 1 and 3, but that we have no information as to whereabouts in this interval its exact value is to be found. The Principle of Indifference would allow us to assume that it is as likely to lie between 1 and 2 as between 2 and 3; for there is no reason for supposing that it lies in one interval rather than in the other. But now consider the specific density. The specific density is the reciprocal of the specific volume, so that if the latter is v the former is $\frac{1}{v}$. Our data *remaining as before, we know that the specific density must lie between 1 and $\frac{1}{3}$, and, by the use of the Principle of Indifference as before, that it is as likely to lie between 1 and $\frac{2}{3}$ as between $\frac{2}{3}$ and $\frac{1}{3}$ ... It follows, therefore, that the specific volume is as likely to lie between 1 and $1\frac{1}{2}$ as between $1\frac{1}{2}$ and 3; whereas we have already proved, relatively to precisely the same* data, *that it is as likely to lie between 1 and 2 as between 2 and 3.*

Reformulations of the principle

- After presenting these and other problems with the Principle of Indifference, Keynes proposed a reformulation of the principle that was intended to avoid the problems (pp. 52–64).
- Keynes's reformulation is complex, not perfectly clear, and does not appear to succeed in avoiding all the contradictions. Therefore, I won't present it.
- More recently, Jaynes has attempted to reformulate the principle, but again without avoiding all contradictions, it seems.
- In short, no formulation of the Principle of Indifference has yet been found that makes it a correct general principle for determining when ip's are equal.

Argument against logical probability

Some philosophers say the difficulties with the Principle of Indifference show that logical probability "does not exist" (van Fraassen) or that "the logical interpretation ... does not allow numerical probabilities" (Gillies).

Refutation of this argument

If "logical probability" means probability that is logical in Carnap's sense, then neither of these conclusions follows.

- Suppose that a probability function p is defined by specifying its numeric values. This function is logical in Carnap's sense.
- The existence of these logical probabilities is guaranteed by the definition of p, which makes no reference to the Principle of Indifference.
- Hence the existence of numerical logical probabilities doesn't depend on the Principle of Indifference.

Ip without the Principle of Indifference

- If p is a stipulatively defined probability function and $p(H|E) = 1/2$, then this identity holds in virtue of the definition of p, not in virtue of the Principle of Indifference.
- Similarly, WB is true in virtue of the concept of ip, not in virtue of the Principle of Indifference. Likewise for other ip's.
- The concept of ip is fixed and learned by examples of its use, not by being given a general principle about ip, such as the Principle of Indifference.
- This is the normal situation for concepts of ordinary language.

Solution of the contradictions

- *Contradictories:* It seems clear that ip(red) = ip(blue) = ip(black), and there are other colors too. Hence ip(red) $< 1/3$ and ip(red) $<$ ip(not-red). *So we can have* $ip(A) \neq ip(\sim A)$ *given no evidence.*
- *Specific volume:* The fact that numerical probabilities can be assigned in different ways that seem equally plausible is reason to think that the ip's here lack definite numerical values. *So no measure correctly represents ip.*

The rule of succession revisited

1. The rule implies that if we have no information at all about an event then the probability of the event is $1/2$. As Keynes showed, this is inconsistent. Hence the rule of succession is inconsistent.
2. Laplace's derivation of the rule assumed that the event has a pp, but not all events have a pp.
3. Even if the event does have a pp, the ip of these pp's need not be uniform on $[0, 1]$; it won't be if the a priori ip of the event isn't $1/2$. E.g., if we are drawing balls from an urn with replacement, there is a pp that the ball will be red, but the a priori ip over these pp's will be weighted towards 0, not flat on $[0, 1]$.
4. There is arguably no precise a priori ip distribution over the pp's.

Questions

5. Show that application of the Principle of Indifference to contradictories gives inconsistent probability assignments.
6. Show that application of the Principle of Indifference to specific volume and density gives inconsistent probability assignments.
7. If there is no sound formulation of the Principle of Indifference, does it follow that quantitative logical probabilities (in Carnap's sense) don't exist? Justify your answer.
8. Show that the Rule of Succession is inconsistent. State two criticisms of Laplace's derivation of this rule.

References

- Maher, Patrick. 2006. "The Concept of Inductive Probability," *Erkenntnis* 65, 185–206. Available online with a uiuc connection. Section 3.1 contains my argument for the existence of ip's, Section 3.2 discusses Ramsey's criticism of Keynes in further detail, and Section 3.3 discusses the Principle of Indifference and gives references.
- Ramsey, Frank P. 1926. "Truth and Probability," Electronic Edition.