Lecture 4
Explication of Physical Probability

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Introduction

- "Physical probability" is the non-logical sense that the word "probability" has in ordinary language (see Lecture 1).
- This lecture will discuss how to explicate this concept.
- Abbreviation: "pp" for physical probability.

Other terms for pp

- *Objective probability* (Howson and Urbach, Gillies)
  - Since inductive probability is also objective, this is poor terminology.
- *Chance* (Levi, Lewis, Mellor, Skyrms)
  - In ordinary language, "chance" is interchangeable with "probability," so this uses "chance" in a special sense.
  - These authors don't distinguish explicandum from explicatum, so their "chance" is not clearly what I mean by "pp."

Compatibility with determinism

Example

Ordinary language statement:

The probability of heads on a toss of this coin is 1/2.

This relates three things:

- An **experiment**: Tossing this coin.
- An **outcome**: This coin landing heads.
- A **number**: $1/2$.

General form of pp statements

The pp of experiment X having outcome O is r (a number).

Abbreviation: $pp_X(O) = r$.

Types versus tokens

- An *experiment token* is a particular event at a particular place and time; it is unrepeatable. E.g., Maher's tossing of a quarter at 1:04 PM today.
- An *experiment type* is a kind of event, e.g., tossing this coin. There can be many tokens of the same type.
- Similarly for outcomes: We can distinguish *tokens* and *types* of outcomes.
- A pp statement relates *types* of experiments and outcomes, not tokens. E.g.,
  - X = tossing this coin (a type)
  - O = this coin landing heads (a type)
  - $r = 1/2$.

One token belongs to many types

A particular (token) coin toss belongs to the following types:

- X: Toss of this coin.
- X': Toss of this coin, starting with the coin in such-and-such a position, with such-and-such a force applied at such-and-such a point, etc.

If $O =$ heads, then $pp_X(O) = 1/2$ but $pp_{X'}(O) = 0$ or 1.

In general: pp is different for different X, even though the different X may share some tokens.

Determinism and pp

- Determinism is the view that the state of the world at one time, together with the laws of nature, completely determine the state of the world at all later times.
- Many philosophers say that if determinism is true, then all pp's are 0 or 1 (Laplace, Lewis, Giere, Mellor).
- But the concept of pp isn't like that.
  - We attribute intermediate pp's in games of chance, while believing that the underlying processes are deterministic.
  - Scientific theories in statistical mechanics, genetics, and social sciences postulate non-extreme pp's in situations where the underlying laws are believed to be deterministic.
- How this is possible:
  - pp relates experiment and outcome *types*, not tokens.
  - Determinism only implies that if X were sufficiently specific, then $pp_X(O)$ would be 0 or 1.
  - But X need not be this specific, and hence there can be more than one possible outcome of X.

Frequency theory

Background

- Let $n =$ the number of observed occurrences (tokens) of X.
- Let $m =$ the number of these occurrences that give O.
- In many cases, when X is repeated, the occurrences of O are random, but m/n eventually settles down near some value (e.g., 1/2 for tossing a normal coin hundreds of times).

Statement of the theory (my formulation; based on Mises 1957)

Define a function $f_X(O)$ as follows:

> $f_X(O) = r$ iff, in an infinite sequence of repetitions of X, O would occur randomly but m/n would approach r as a limit.

The theory proposes $f_X(O)$ as an explicatum for $pp_X(O)$.

Criticism of the frequency theory

Consider a simple example, e.g., X is tossing a particular coin and O is the coin landing heads. We can suppose $pp_X(O) = 1/2$. But:

1. O may occur or not on each trial of X, so the occurrences of O need not be random. Hence it is not true that O *would* occur randomly.
2. Similarly, it is not true that m/n *would* approach a limit; it need not.
3. Similarly, if it did approach a limit, there is no value that the limit *would* be; it could be any number from 0 to 1.

For all these reasons, $f_X(O)$ does not exist. Therefore, f is not similar enough to pp to be able to be used in place of pp, and hence is a poor explicatum.

Questions

1. What is an experiment token? What is an experiment type? Give an example of each.
2. What is determinism? Is it possible for pp's to have values other than 0 or 1 if determinism is true? Explain.
3. Suppose $f_X(O)$ is defined as follows:

   > $f_X(O) = r$ iff, in an infinite sequence of repetitions of X, O would occur randomly but m/n would approach r as a limit.

   Is $f_X(O)$ a good explicatum for $pp_X(O)$? Why, or why not?

Propensity theory

- We've seen that the frequency theory is an unsatisfactory attempt to explicate pp.
- An alternative is a *propensity theory*: pp is a propensity or tendency of the experiment to give particular outcomes; frequencies are evidence of this propensity but don't define it.
  - E.g., tossing a normal coin has a propensity of 1/2 to give heads.
- *Objection:* "propensity" and "tendency" are too vague.
- *Solution:* Introduce an explicatum q for pp by stating postulates that relate q to p (an explicatum for inductive probability).
  - The meaning of q is given by these postulates.
  - A term introduced by postulates is called a "theoretical term."

Direct inference postulate

Terminology

- A *Q-proposition* is a consistent conjunction of propositions of the form $q_X(O) = r$.
- "Xa" means a is a token of X.
- "Oa" means a has outcome O.

In what follows, Q is always a Q-proposition and a is a token event.

Direct inference postulate (DI)

If Q implies that $q_X(O) = r$, then $p(Oa|Xa.Q) = r$.

Since p is logical in Carnap's sense, DI is analytic; it is part of the definition of p.

Proof that q obeys the laws of probability

From DI, and the fact that p obeys the laws of conditional probability, it follows that q_X obeys the laws of unconditional probability (with predicates in place of sentences). Proof:

K1: $q_X(O) \geq 0$

Let Q state the true value of $q_X(O)$. Then:

$$\begin{aligned} q_X(O) &= p(Oa|Xa.Q), \text{ by DI} \\ &\geq 0, \text{ by A1.} \end{aligned}$$

K2: If O is analytic, $q_X(O) = 1$

Suppose O is analytic. Let Q state the true value of $q_X(O)$. Then:

$$\begin{aligned} q_X(O) &= p(Oa|Xa.Q), \text{ by DI} \\ &= 1, \text{ by P1 corollary 1.} \end{aligned}$$

K3: If $O_1.O_2$ is contradictory, $q_X(O_1 \vee O_2) = q_X(O_1) + q_X(O_2)$

Suppose $O_1.O_2$ is contradictory. Let Q state the true values of $q_X(O_1)$, $q_X(O_2)$, and $q_X(O_1 \vee O_2)$. Then:

$$\begin{aligned} q_X(O_1 \vee O_2) &= p(O_1 a \vee O_2 a | Xa.Q), \text{ by DI} \\ &= p(O_1 a | Xa.Q) + p(O_2 a | Xa.Q), \text{ by P4} \\ &= q_X(O_1) + q_X(O_2), \text{ by DI.} \end{aligned}$$

Irrelevance postulate

- The mere fact that an experiment is performed is not itself evidence regarding the pp of any outcome of that experiment.
- E.g., that a coin is tossed does not by itself give any evidence about the pp of the coin landing heads when tossed.

For any A, let $p(A)$ be an abbreviation for $p(A|T)$, where T is a tautology. The above observation motivates adopting:

Irrelevance postulate (IR)

$p(Q|Xa) = p(Q)$.

This is part of the definition of p and hence is analytic.

Learning the values of q

No postulate specifies the values of $q_X(O)$ for contingent O. Thus q isn't a logical concept and we need to learn its values empirically.

How this is done

1. Define a priori values $p(Q)$ for Q-propositions.
2. Use DI, IR, and Bayes's theorem to determine $p(Q|E)$ for evidence E.

Bayes's theorem

If $p(H_1 \vee \cdots \vee H_n) = 1$ and $p(H_i.H_j) = 0$ for $i \neq j$ then

$$p(H_i|E) = \frac{p(E|H_i)p(H_i)}{p(E|H_1)p(H_1) + \cdots + p(E|H_n)p(H_n)}.$$

Example

Let X = a particular die is tossed; O = it comes up six. Let Q_f (fair) mean that $q_X(O) = 1/6$ and Q_b (biased) mean that $q_X(O) = 1/2$. Suppose $p(Q_f) = p(Q_b) = 1/2$. What is $p(Q_b|Xa.Oa)$?

$$
\begin{aligned}
p(Q_b|Xa.Oa) &= \frac{p(Oa|Xa.Q_b)p(Q_b|Xa)}{p(Oa|Xa.Q_b)p(Q_b|Xa) + p(Oa|Xa.Q_f)p(Q_f|Xa)}, \\
&\qquad\qquad \text{by Bayes's theorem} \\
&= \frac{(1/2)p(Q_b|Xa)}{(1/2)p(Q_b|Xa) + (1/6)p(Q_f|Xa)}, \text{ by DI} \\
&= \frac{(1/2)(1/2)}{(1/2)(1/2) + (1/6)(1/2)}, \text{ by IR} \\
&= 3/4 > p(Q_b).
\end{aligned}
$$

Independence postulate

- I think there should be a further postulate which says that, when q_X exists, the outcomes of repeated tokens of X are independent.
  - E.g., if X is tossing a particular coin, X^2 is tossing it twice, and H is the coin landing heads, $q_{X^2}(HH) = [q_X(H)]^2$.
- It can then be proved that, when q_X exists, any possible outcome O of X will probably occur randomly, and the relative frequency of O will probably be close to $q_X(O)$.
- These probabilities approach 1 as the number of repetitions approaches infinity.
- This is like what the frequency theory says, except that probability 1 doesn't mean it must happen.

Existence of physical probabilities

It is widely agreed that pp's don't always exist. According to the explication given here:

1. Pp is a relation between an experiment type and an outcome type. Other things don't have pp's.
   - Example: There is not a pp that the extinction of the dinosaurs was caused by a large asteroid striking the earth.
2. If X and O are experiment and outcome types (respectively), there isn't a pp of X having outcome O if such a pp would not satisfy all the postulates.
   - Example: Let $X =$ an ordinary coin is turned over, $O =$ the coin has heads facing up. A pp of X having outcome O wouldn't satisfy the independence postulate, so there is no such pp.

Questions

4. What is a Q-proposition? What is the principle of direct inference? What is the principle of irrelevance?
5. State Maher's proof of the following:
   (a) $q_X(O) \geq 0$.
   (b) If O is analytic, $q_X(O) = 1$.
   (c) If $O_1.O_2$ is contradictory, $q_X(O_1 \vee O_2) = q_X(O_1) + q_X(O_2)$.
6. According to Maher's explication, how do we learn the values of physical probabilities? (A brief verbal answer is sufficient.)
7. Is there is a physical probability that Cleopatra's death was caused by snake bites, according to Maher's account of physical probability? Explain.
8. Let X = a day's maximum temperature is recorded, O = the recorded temperature exceeds 50 degrees. Is there a physical probability of X having outcome O, according to Maher's account of physical probability? Explain.

References

- Isaac Levi, "Chance," *Philosophical Topics 18* 2 (Fall 1990), 117–149. Reconciles pp with determinism in essentially the same way I have done (though with different terminology).
- David Lewis, "A Subjectivist's Guide to Objective Chance," reprinted with postscripts in Lewis's *Philosophical Papers* vol. 2, Oxford U.P. 1986. Takes pp to be relative to the whole prior history of the world, rather than to types of experiments, and hence asserts that pp is incompatible with determinism.
- Richard von Mises, *Probability, Statistics and Truth*, 2nd English edition, George Allen & Unwin 1957. Dover reprint. A classic and readable presentation of the frequency theory.